# Discrete Random Variables

Overview of each discrete random variable

- The Bernoulli random variable is one that has only two possible outcomes: success or failure, and we count the number of successes. Any experiment where we only consider two possible outcomes is called a Bernoulli trial. It has one parameter $p$, the probability of a success. The easiest example is a coin flip.
- A binomial random variable is one where you perform exactly $n$ independent Bernoulli trials, and we count the number of successes. It has two parameters $n$ and $p$, where $n$ is the number of Bernoulli trials, and $p$ is the probability of success. Binomial can be modeled by $n$ coin flips.
- A geometric random variable is when we perform Bernoulli trials over and over until the first success, and we count the number of trials it takes. It has one parameter $p$, the probability of a success. In this case, we flip a coin over and over until we get heads, and we count the number of coin flips it takes to get our first head.
- A negative binomial random variable is one where we perform independent Bernoulli trials over and over until we get your $r$th success, and we count the total number of trials it takes to reach that point. It has two parameters $r$ and $p$, where $r$ is the number of desired successes, and $p$ is the probability of success.
- A hypergeometric random variable is one where you have a sample of size $B + G$, that has 2 types, $B$ and $G$, and you select a sample of size $n$. It has three parameters, $B, G, n$, where $B$ is the number of objects of type $B$, $G$ is the number of objects of type $G$, and $n$ is the number of items in the sample drawn. We count the number of objects of type $B$.
- A Poisson random variable is one used to model the number of times an event occurs in an interval of time or space. It has one parameter $\lambda$. Examples include
  - The number of mistakes on a page of a book
  - The number of patients arriving in an emergency room between 10 and 11 pm
  - The number of emails you get in a day
  - The number of radioactive particles that is released from a sample in a second.

**Remark.** *The difference between the binomial random variable and the negative binomial random variable is the following. The binomial random variable fixes the number of Bernoulli trials performed, and we count the number of successes. For the negative binomial, we fix the number of successes and perform Bernoulli trials until we get those successes, counting the number that it takes. For example, a binomial case is when we flip a coin 5 times and count the number of heads. For the negative binomial, we flip a coin until we get 5 heads and count the total number of coin flips it takes.*

# Problems in Discrete Random Variables

In the problems below, before answering the question, state which random variable to use, why we should be using this random variable, and the parameters of the random variable.

1. A certain typing agency employs 2 typists. The average number of errors per article is 3 when typed by the first typist, and 4.2 when typed by the second. If an article is equally likely to be typed by either typist, what is the probability that it will have no errors?
2. Suppose that a batch of 100 items contains 6 that are defective and 94 that are not defective. If $X$ is the number of defective items in a randomly drawn sample of 10 items from the batch, find $P(X = 0)$ and $P(X < 2)$.

3. A multiple choice test has 100 questions, and each question has 5 choices. What is the probability that if you guess every question, and each guess is independent of each other, that you get 10 questions correct?

4. Suppose that in order to qualify to be an actuary you have to pass a test three times, and that your probability of passing any one test is 0.6. You are allowed to take it as many times as you wish. What is the probability that you would need to take it at least five times before you become an actuary? What is the expected number of tests you have to take before you become an actuary? What is the variance of the number of tests you need to take?

5. A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability .2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority decoding," that is, a mix of 0's and 1's will be interpreted as the number that shows up more often. For example, 00100 and 01010 will be interpreted as a 0, 11011 and 11001 will be interpreted as 1. What is the probability that the message will be wrong when decoded?

6. Suppose that, on average, 3.2 alpha particles are given off in a 1 second interval by 1 gram of radioactive material. What is the probability that no more than 2 alpha particles will appear in 1 second by this material?

7. Two teams play a series of games, the first team to win 4 games is the winner overall. Suppose that one of the teams is stronger and has probability 0.6 to win each game, independent of any other games. What is the probability that the stronger team wins in 4 games? 5 games? 6 games? 7 games?

8. Two teams play a series of games, best 2 out of 3. Suppose that one of the teams is stronger and has probability 0.6 to win each game, independent of any other games. What is the probability the stronger team wins overall?

9. A fair coin is continually flipped until heads appears for the 10th time. Let $X$ denote the number of tails that occur. Compute the probability mass function of $X$.

10. Suppose that the average number of cars abandoned weekly on a certain highway is 2.2. Approximate the probability that there will be no abandoned cars in the next week. Approximate the probability that there will be at least 2 abandoned cars in the next week.

11. An urn contains 4 white and 4 black balls. We randomly choose 4 balls. What is the probability that the balls we have chosen contain 2 white balls and 2 black balls?

12. An urn contains 4 white and 4 black balls. We randomly choose 4 balls. If we get 2 white and 2 black balls, we stop. If not, we replace the balls in the urn and again randomly select 4 balls. This process continues until we get 2 white balls and 2 black balls. What is the probability that it takes exactly $n$ selections?

13. A man claims to have extrasensory perception. As a test, a fair coin is flipped 10 times, and the man is asked to predict the outcome in advance. He gets 7 out 10 correct. What is the probability that he would have done at least this well if he did not have ESP?

14. You take a test until you pass it. Suppose that your probability of passing is independent of previous attempts, and the probability of passing it is 0.2. What is the expected number of tries until you pass it? What is the standard deviation of the number of trials?

15. The monthly worldwide average number of airplane crashes of commercial airlines is 3.5. What is the probability that there will be at least 2 such accidents in the next month? What is the probability that there will be at most 1 accident in the next month?